Practice Optimization and Related Rates questions

82. If $y = 2x - 8$, what is the minimum value of the product xy ?

(A) -16 (B) -8 (C) -4 (D) 0 (E) 2

1990 BC3

Let $f(x) = 12 - x^2$ for $x \geq 0$ and $f(x) \geq 0$.

(a) The line tangent to the graph of f at the point $(k, f(k))$ intercepts the x-axis at $x = 4$. What is the value of k ?

(b) An isosceles triangle whose base is the interval from $(0,0)$ to $(c,0)$ has its vertex on the graph of f. For what value of c does the triangle have maximum area? Justify your answer.

AP® CALCULUS AB
2009 SCORING GUIDELINES (Form B)

Question 1

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\,dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

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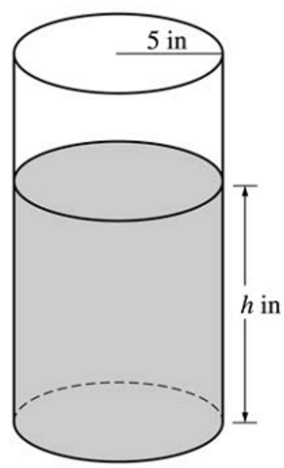
2003 SCORING GUIDELINES

Question 5

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)



(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

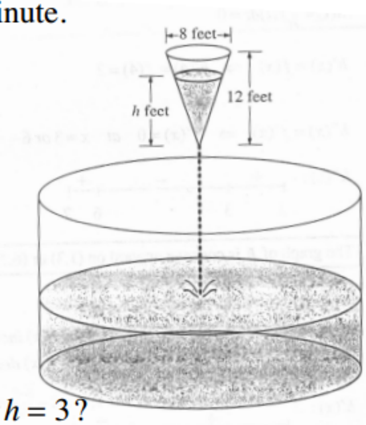
(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

(c) At what time t is the coffeepot empty?

AP 1995 – AB 5, BC 3

As shown in the figure, water is draining from a conical tank with height 12 feet and diameter of 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at a rate of $(h-12)$ feet per minute.

The volume V of a cone with radius r and height h is $V=\frac{1}{3}\pi r^2 h$.



a. Write an expression for the volume of the water in the conical tank as a function of h.
b. At what rate is the volume of water in the conical tank changing when $h=3$? Indicate units of measure.
c. Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h=3$? Indicate units of measure.

1990 AB4

The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second. (Note: The volume of a sphere with radius r is $V=\frac{4}{3}\pi r^3$.)

(a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?

(b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

(c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?